This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 1, and 2) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2011 yearly problem is in the "Solutions" section.

PROBLEMS

Y2012. How many integers from 1 to 100 can you form using the digits 2, 0, 1, and 2 exactly once each; the operators +, −, × (multiplication), and / (division); and exponentiation? We seek solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 1, 2 are preferred. Parentheses may be used; they do not count as operators. A leading minus sign, however, does count as an operator.

J/F 1. Longtime Puzzle Corner contributor Frank Rubin has created a new class of grid base arithmetic puzzles and has a set up a website with expanded rules, hints, etc. To check them out, see sumsumpuzzle.com/sumsum.htm.

The 10-by-10 grid below has a descriptive border column and row on the left and top, respectively. The problem is to black out two nonadjacent squares in each column and row and then write the digits from 1 to 8 once each in the eight remaining squares. Each run of adjacent digits must add up to the corresponding sum shown to the left or on top. If neither black square is at the end of the row or column, there are three runs; if one is at the end, there are two. If both are at ends, there would be just one run, but this does not occur in the problem below.

	6 2 28	31 5	22 14	12 20 4	19 8 9	15 21	8 28	8 15 13	12 8 16	11 23 2
13, 8, 5										
27, 9										
11, 25										
29, 7		8				4				
13, 5, 18										
26, 5, 5										
8, 18, 10										
8, 5, 23										
16, 20										
1, 18, 17										

J/F 2. Glen Case doesn't think of tennis or bridge when he achieves a set. Instead, he's referring to the card game SET, in which each card has four features. (In the game, the features are shape, color, shading, and number. See www.setgame.com for more information.) Each feature can have one of three states. Thus the deck is made of $3^4 = 81$ distinct cards. A "set" is a group of exactly three cards for which each feature is either in a single state or in three different states.

Chase wonders, given some number N of randomly selected SET cards, what is the probability of there being no "set"? For N=1 and N=2, for example, the probability is of course one–three cards are needed for a set. For N=3 the probability is 78/79–given any two cards there is only one other card that will complete the "set."

J/F 3. Presumably in deference to my "second career" in computer science, Jerry Grossman sends us a "buggy" problem from Laci Babai, who learned it growing up in Hungary.

Maria is terrified of ticks and wants to create a structure to prevent the infinitesimally small tick in her bedroom from crawling on her while she is asleep. The tick is somewhere on the walls or ceiling of her room. The tick can crawl on any surface except water and can drop vertically from any point. Maria puts each leg of her bed into a bowl of water to stop the tick from crawling up the legs. She then constructs another structure and happily sleeps through the night. What is it? The only constraint on the structure is that Maria and the tick remain in the same connected component of the air space.

SPEED DEPARTMENT

Oren Helbok wonders: what is a mathematician's favorite percussion instrument?

SOLUTIONS

Y2011. How many integers from 1 to 100 can you form using the digits 2, 0, 1, and 1 exactly once each; the operators +, −, × (multiplication), and / (division); and exponentiation? We seek solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 1, 1 are preferred. Parentheses do not count as operators. A leading minus sign does count as an operator.

$1 = 1^{201}$
$2 = 12 - 10$
$3 = 11^0 + 2$
$4 = 10/2 - 1$
$5 = 10/2 \times 1$
$6 = 10/2 + 1$
$7 = 10 - 2 - 1$
$8 = 10 - 2^1$
$9 = 20 - 11$
$10 = 20/(1 + 1)$
$11 = 21 - 10$
$12 = 2^0 + 11$
$13 = 2 + 0 + 11$
$18 = 20 - 1 - 1$
$19 = 20 - 1^1$
$20 = 20^{1^1}$
$21 = 20 + 1^1$
$22 = 10 + 12$
$30 = 10 \times (2 + 1)$
$31 = 20 + 11$
$40 = 20 \times (1 + 1)$
$55 = 110/2$
$81 = (10 - 1)^2$
$99 = 101 - 2$
$100 = 10^{2^1}$

The preceding solution, from John Chandler, suggests that only 25 numbers can be expressed. The situation should improve in 2013, when at least there are no repeated digits. If we hang on until 2134, there will be no 0s and no repeats!

S/O 1. I know that the shortest chess game ending in checkmate is the two-move "fool's mate" (1. f3 e5 2. g4 Qh4) but have never seen the following related question from Sorab Vatcha: What is the shortest chess game ending in stalemate?

Several readers have pointed out that Sam Lloyd has found a 10-move solution (see the Wikipedia entry on stalemate for more details). Timothy Chow sent us the following.

Stalemate can be achieved on White's 10th move, as follows.

1. c4 h5	6. Qxd7+ Kf7
2. h4 a5	7. Qxb7 Qd3
3. Qa4 Ra6	8. Qxb8 Qh7
4. Qxa5 Rah6	9. Qxc8 Kg6
5. Qxc7 f6	10. Qe6 stalemate

This sequence of moves has actually occurred in a tournament game! In a 1995 Swedish tournament for juniors, Johan Upmark and Robin Johansson agreed ahead of time to a draw. When the time came for their game, they played out the moves of Lloyd's game.

Chess-problem specialists have noted that Lloyd's game contains an aesthetic flaw in that the sequence of 10 moves is not unique; for example, White can transpose the order of his first two moves. In a 2002 issue of *StrateGems* magazine, Radovan Tomasevic and Kostas Prentos published a variant of Lloyd's position, shown here, that is not as short as Lloyd's game (White delivers stalemate on White's 12th move) but has the virtue of a unique solution. You might enjoy reconstructing the sequence of moves leading up to Tomasevic and Prentos's position. (Uppercase denotes White; lowercase denotes Black.)

.	.	.	.	.	b	n	r
.	.	.	.	p	.	p	b
.	.	.	.	Q	p	k	r
.	.	.	.	.	.	.	p
.	.	.	.	.	.	.	P
.	.	.	.	.	.	.	.
P	P	.	P	.	P	P	.
R	N	B	.	K	B	N	.

S/O 2. Jerry Grossman wonders if there exists an infinite number of sets such that the intersection of every two distinct sets in the collection is nonempty, but the intersection of every three sets in the collection is empty.

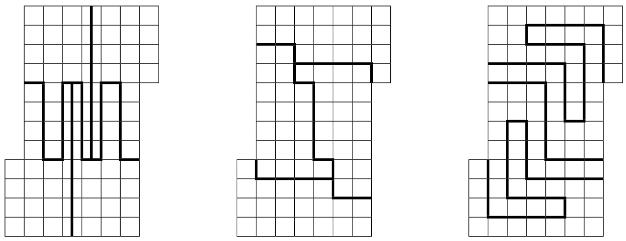
This was a quite a popular problem, and I received many fine solutions. I must confess that when I first saw the problem, I hadn't imagined that such simple, elegant solutions would be possible.

Ken Ziegler notes that we should have said "every three *distinct* sets." He solves the corrected problem as follows.

For each positive integer n let $S(n)$ be the collection of all sets $\{i, j\}$, where i and j are distinct positive integers, one of which is n. If n and m are distinct, then $S(n)$ and $S(m)$ each contain $\{n, m\}$, so their intersection is nonempty. But no set $\{n, m\}$ can lie in $S(a)$, $S(b)$, and $S(c)$ if a, b, c are distinct since at least one of a, b, c does not lie in $\{n, m\}$. Thus, $S(a)$, $S(b)$, and $S(c)$ have an empty intersection.

S/O 3. In Solomon Golomb's October 1987 installment of "Golomb's Gambits," we are asked to dissect the figure below into four congruent pieces. These four pieces do *not* have to be similar to the original.

Perhaps to balance the avalanche of solutions to S/O 2, I have only the proposer's solutions to this problem. The only other response received did not have four *congruent* pieces.



BETTER LATE THAN NEVER

2011 M/J 3. Robert Ackerberg notes that there was a typo in the final subtraction; the correct answer is 89°47'10".

OTHER RESPONDERS

Responses have also been received from M. Badavam, P. Belmont, M. Bolotin, R. Botsford, J. Bross, T. Chow, F. Cornelius, B. Currier, C. Dale, D. Detlefs, M. Eiger, R. Ellis, J. Feil, M. Finkelstein, P. Fiore, B. Godfrey, R. Golomb, M. Gordy, K. Hanf, J. Harmse, S. Howlett, J. Karnofsky, K. Kelley, P. Kramer, T. Krichner, A. Kunin, E. Levin, D. Linden, V. Luchangco, T. Mita, M. Perkins, S. Resnikoff, B. Rothleder, J. Russell, A. Sahai, S. Scheinberg, T. Schonbek, J. Schwartz, E. Sheldon, E. Signorelli, J. Sokol, E. Staples, W. Stein, E. Turner, S. Vatcha, L. Wagner, A. Wasserman, C. Wiegert, and K. Zeger.

PROPOSER'S SOLUTION TO SPEED PROBLEM

Pascal's triangle. ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.