This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 1, and 0) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2009 yearly problem is in the "Solutions" section.

PROBLEMS

Y2010. How many integers from 1 to 100 can you form using the digits 2, 0, 1, and 0 exactly once each; the operators +, −, × (multiplication), and / (division); and exponentiation? We seek solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 1, 0 are preferred. Parentheses may be used and do not count as operators. A leading minus sign does count as an operator. Zero to the zero power is *not* permitted, but 00 can be used for zero.

J/F 1. There was an unfortunate typo in 2009 S/O1 (see the solutions section below), so we reopen the problem here as 2010 J/F1. Jorgen Harmse, inspired by Frank Schuld's book *The Simple Squeeze in Bridge*, asks us the following. South is declarer at 4NT.

	♠6 5 4 3 2	
	♠A K 4	
	♠Q	
	♠A 8 5 3	
♠Q J 10 8 7		♠–
♠Q 10 9 7 3		♠J 8
♠A 8		♠9 7 6 4 2
♠J		♠Q 10 9 7 4 2
	♠A K 9	
	♠6 5 2	
	♠K J 10 5 3	
	♠K 6	

How can he make the contract against a spade queen lead? Can he make it against a heart lead? In both cases assume double dummy (all hands visible, best play on both sides).

J/F 2. Anthony Benis offers us "The Fork in the Road."

You arrive in the strange land of Qralaak. There are only two personality types there: TTTs, who always tell the truth, and FFFs, who always lie. Also, there are only two cities: Truthtown, populated only by TTTs, and Othertown, populated by both types.

In urgent need of directions to Othertown, where you can buy life-saving medicine, you arrive at a fork in the road where two strangers are standing and chatting. One has just arrived from Truthtown, and one has arrived from Othertown.

One of the roads at the fork ("left" or "right") goes to Truthtown and the other goes to Othertown, but you do not know which is which. You also know nothing about the two strangers (not even whether each knows what type the other is), except that one came from Truthtown and one came from Othertown.

Truthtowners and Othertowners live in strange isolation: although intelligent, they are ignorant, and you cannot assume that either stranger is aware of the location, or existence, of the other town … or even knows that there are only two towns.

The two strangers reluctantly allow you to ask a single question, which each will answer with "Yes" or "No." The single question must be must be stated so that each individual can give a clear "Yes" or "No" answer. They will not say anything except "Yes" or "No" once.

The question must be reasonably short—say, under 50 words.

SPEED DEPARTMENT

In calculus, John Prussing recalls, he was taught that where the second derivative is zero, the function has a point of inflection. He wants you to find a point with zero second derivative that is, instead, a minimum.

SOLUTIONS

Y2009. The following solution merges the results from Ermanno Signorelli and Alan Taylor.

$1 = 209^0$	$11 = 20 + 0 - 9$	$70 = 90 - 20$
$2 = 2 + 0 \times 90$	$18 = 2 \times 9 + 00$	$81 = 9^2 + 00$
$3 = 2 + 90^0$	$19 = 20 - 9^0$	$88 = 90 + 0 - 2$
$7 = 9 - 2 + 00$	$20 = 20 + 0 \times 9$	$89 = 90 - 2^0$
$8 = 9 - 20^0$	$21 = 20 + 9^0$	$90 = 2 \times 0 + 90$
$9 = 20 \times 0 + 9$	$29 = 29 + 00$	$91 = 2^0 + 90$
$10 = 20^0 + 9$	$45 = 90/2 + 0$	$92 = 92 + 00$

S/O 1. As I mentioned above, this problem was incorrectly stated in the September/October issue, and the corrected version appears in this issue as J/F1.

S/O 2. Ermanno Signorelli observes that given a triangle ABC with a length of BC, one can construct a circle with center A and radius a. Similarly for B with radius b (the length of AC) and for C with radius c (the length of AB). How, he asks, can one construct an exterior circle tangent to the other three?

The following solution is currently anonymous. It looks as though it is a printout of a PDF file; it is very possible that I misplaced the cover letter. If the author lets me know, I will give credit in a future issue. This problem must have been difficult, because I have no other solution.

The centers of two tangent circles are collinear with the point of tangency. If the circle to construct has center O and radius r, it follows for the implicit line through A that $r = a + \text{AO}$, and similarly for the other two corners. If we can construct r, then we can find O as the intersection of the circle of radius $r - a$ centered at A with the corresponding circles at B and C. Hence it suffices to solve for r algebraically and then give a construction for it.

Writing O as (x, y) and A as (A_x, A_y), squaring $AO = r - a$ gives E(a): $(x - A_x)^2 + (y - A_y)^2 = (r - a)^2$ and corresponding equations E(b) and E(c). By inspection, E(a) – E(b) and E(a) – E(c) are linear equations in x, y, and r. Using these to eliminate x and y in E(a) gives a quadratic equation in r. In particular, there will be two solutions for r, although one could be spurious because E(a) involved squaring.

Solving the equations as above leads to a complicated-looking formula. By symmetry, r should be expressible in terms of only a, b, and c. After some rearrangement we get $r = s + \frac{d^2}{e^2/(2s) \pm 2d}$ with $s = ½(a+b+c)$, $d = \sqrt{(s-a)(s-b)(s-c)/s}$, and $e = \sqrt{s^2 - (s-a)^2 - (s-b)^2 - (s-c)^2}$. As long as $e^2/(2s) \pm 2d$ is positive, r will be a solution to the problem. This always holds for the $+2d$ form.

Now d is the radius of the inscribed circle in ABC, which is easy to construct using the fact that the center of this circle is the intersection of the angle bisectors. We can construct e using an appropriate sequence of right triangles. A value like x^2/y can be constructed using similar triangles. Combining these with other simple constructions gives a construction for r.

Note that the center of the $+2d$ solution is always inside the triangle. The $-2d$ solution is always associated with some circle tangent to the drawn ones, with center outside the triangle.

S/O 3. Howard Cohen served up the following. If a tennis player has probability p of winning each point on her serve, what is the probability that she will win her service game?

Tim Soncrant did a statistical analysis, and as a check, he and his son Andrew developed a random-number-based algorithm to simulate groups of 100 million games played. Groups were simulated for many values of p (from 0 to 1) and yielded winning probabilities in agreement with the proposed solution. His analysis follows.

We are asked to find the probability P that a tennis game will be won, given the probability p of winning each point. Winning a game requires winning at least four points, and at least two more than the opponent.

For S_4, S_5, and S_6 the probability of winning (W) or losing (L) the game on that point is the probability of having won (or lost) exactly three of the preceding points and the current point.

Point	*W*	*L*
S_4	p^4	$(1-p)^4$
S_5	$4p^4(1-p)$	$4p(1-p)^4$
S_6	$10p^4(1-p)2$	$10p^2(1-p)^4$

Note: For W_5 and L_5 there are four possible scenarios; for W_6 and L_6 there are 10.

If after six points the game has not ended, the game must be tied at 3–3 (deuce). Thereafter, the game cannot end on an odd-numbered point and can be won (or lost) only by winning (or losing) a pair, T, of consecutive odd-even points ($S_{7\text{-}8}$, $S_{9\text{-}10}$, ...). Thus, after the first six points there follows an infinite series of point pairs, each of which is progressively less likely to be played. For each pair of points T_i, the contribution to the probability of winning (W_i) or losing (L_i) the game is equal to the probability of winning or losing both points (p^2 and $(1-p)^2$, respectively) times the probability that the pair of points will be played (i.e., the probability that the game is not yet over, Q_i).

The probability that the first pair of points, T_0 ($S_{7\text{-}8}$), will be played (Q_0) is equal to the probability that the game was not over after the sixth point: $Q_0 = 1 - W_4 - L_4 - W_5 - L_5 - W_6 - L_6$.

For subsequent pairs $T_{i=1}\infty$, the probability that each will be played is progressively reduced by the probability that the game was won on a prior pair, according to the formula $Q_i = Q_0 \times (1 - p^2 - (1-p)^2)^i$.

The corresponding probability that the game will be won on any pair of pair of points T_i is equal to $W_i = p2 \times Q_i$.

For the infinite series of pairs, W_i converges and the sum of the series is equal to $\Sigma_{i=0}\infty W_i = p^2 \times Q_0 / (p^2 + (1-p)^2)$.

Thus, the probability of winning the game is equal to $P = W_4 + W_5 + W_6 + \Sigma_{i=0}\infty W_i$, or explicitly, $P = p^4 + 4p^4(1-p) + 10p^4(1-p)^2 + p^2(1 - p^4 - 4p^4(1-p) - 10p^4(1-p)^2 - (1-p)^4 - 4p(1-p)^4 - 10p^2(1-p)^4)/(p^2 + (1-p)^2)$. A more computationally convenient form is $P = (-8p^7 + 28p^6 - 34p^5 + 15p^4)/(2p^2 - 2p + 1)$.

A graph of the results can be found on the Puzzle Corner website: cs.nyu.edu/~gottlieb/tr.

BETTER LATE THAN NEVER

2009 M/J 2. Charles Wampler notes that Bertini, a free software package he and others developed, is useful for solving systems of polynomial equations such as those occurring in this problem.

OTHER RESPONDERS

Responses have also been received from C. Brooks, V. Christensen, H. Cohen, C. Dale, T. Falcone, D. Freeman, K. Haruta, R. Hess, H. Ingraham, M. Kay, J. Kenton, P. Kramer, G. Lesieutre, S. Lippow, D. Mellinger, Z. Moledina, B. Norris, A. Ornstein, R. Peacetree, J. Prussing, K. Rosato, E. Sard, L. Sartori, A. Smith, G. Stith, T. Terwilliger, H. Thiriez, S. Ulens, R. Wake, A. Wasserman, D. Watson, B. Wolf, and K. Zeger.

PROPOSER'S SOLUTION TO SPEED PROBLEM

The point (0,0) for the function $f(x) = x^4$. ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.