Season's greetings from our "Winter Wonderland." Although the solstice was yesterday, our first real snowstorm, and hence the emotional start of winter, was a few days earlier. We received about a foot here in northern Westchester, and the lake, woods, and surrounding area are beautiful to behold, if rather inconvenient to deal with.

PROBLEMS

M/A 1. Our bridgemeister, Larry Kells, wants you to make seven hearts against best defense despite one opponent's holding the J97543 of hearts, a side ace, and a guarded side king. Oh, yes—the other opponent has 10 high-card points.

M/A 2. Avi Ornstein (and his friend Fibo) like to play with sequences. Choose an integer $a \geq 2$ and consider the two sequences

$y_1 = 1 \quad y_2 = a - 1 \quad y_n = a \cdot y_{n-1} - y_{n-2}$

and

$x_1 = 1 \quad x_2 = a \quad x_n = a \cdot x_{n-1} - x_{n-2}$

How are these two sequences related?

M/A 3. We close with another "logical hat" problem from Richard Hess. Recall that in logical hat problems each logician wears a hat with a positive integer on it. All the logicians are error-free in their reasoning and are given this information as well as other information in the problem.

Integers x and $y > 2$, but not necessarily different, are chosen. The number on A's hat is $x \cdot y$ and the number on B's is $x + y$. A and B make statements as follows.

A1: "There is no way you can know the number on your hat."

B1: "I now know my number."

A2: "I now know my number. Both our numbers are less than 500."

What numbers are on A and B?

SPEED DEPARTMENT

John Prussing's hybrid car traveled north on a U.S. interstate highway. Five hours after passing mile marker 100, traveling at a steady 60 miles per hour with no stops, it passes mile marker 250. How is this possible?

SOLUTIONS

N/D 1. Yet another bizarre deal occurred at Larry Kells's duplicate-bridge club. At one table, North-South bid and made seven spades. At another table, on the same deal, East-West bid and made one spade redoubled! Now surely, if one side can make a grand slam in a suit, the other side can't possibly make any contract in the same suit ... can it? Some defender must have made a terrible mistake in play. However, when the scorer asked for verification of the scores, all the players at the two tables involved confirmed that those contracts were made and, furthermore, that no defender ever made any error! Unfortunately, the cards were all mixed up after they were played for the last time. Can you help reconstruct the deal?

Tom Terwilliger submits the following solution and notes that several of the low cards can be exchanged. The key is that West has only black cards and North only red.

North:
♠ –
♥ A to 3
♦ 2
♣ –

West:
♠ J 10 9
♥ –
♦ –
♣ K to 4

East:
♠ 6 5 4 3 2
♥ –
♦ A to 8
♣ 3

South:
♠ A K Q 8 7
♥ 2
♦ 7 6 5 4 3
♣ A 2

"When South declares and West leads, he must lead into South, who can simply draw trump and then run North's hearts for all 13 tricks. When West declares and North leads, he can't reach his partner. His best shot is a diamond. East wins and starts cashing diamonds. South's best play is to discard on tricks six and seven. East must then lead into South, who wins the last six tricks. Should South ruff the sixth diamond low, West will overruff. South then loses the opportunity to draw East's trump and wins only four trump and the ace of clubs; East-West make eight tricks. Should South ruff high, he will similarly take only five tricks. And should North mistakenly lead a heart, East will ruff and run seven diamonds as before, winding up with eight tricks."

N/D 2. Loren Bonderson enjoyed the problem of finding the grazing area of a goat tethered to a silo so much that he has extended it to three dimensions (and moved it from farming to astronautics).

If an astronaut is tethered to a spherical satellite of radius R with a tether of length πR, what is the volume of space that the astronaut may reach?

Michael Brill noticed two unstated assumptions. First, we are ignoring the fact that the satellite is moving. Second, we are not permitting the astronaut to jettison some mass, thus imparting angular momentum. The following solution, complete with diagram, is from Eric Nelson-Melby.

"This is similar to the two-dimensional problem with a goat tied to a point on a silo. The volume accessible is the upper half of the area in the two-dimensional problem, rotated around the axis that runs from the center of the sphere through the tether point. For the region where the tether does not wrap around the sphere at all, the volume is simply that of a hemisphere with radius R:

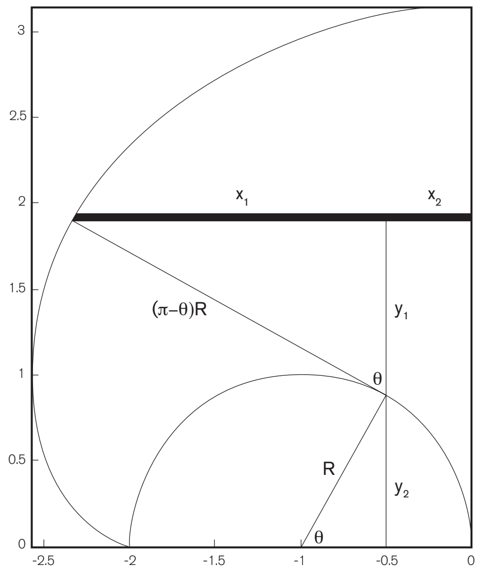


Figure 1

$V_1 = {}^2/_3 \pi^4 R^3$

"The other part of the accessible volume is that of the area shown in Figure 1, rotated about the axis of the small satellite sphere (minus the volume $4/3\pi R^3$ of the satellite). The entire volume, including the satellite, can be calculated with the shell method. Referring to Figure 1, for a cylindrical shell of thickness dy at radius $y = y_1 + y_2$ and height $x = x_1 + x_2$, the volume is

$V_2 = \int_0^{\pi R} dy 2\pi y x$

"Using the angle θ in the figure, the lengths x and y can be expressed as functions of θ, which ranges from π to 0 as y goes from 0 to πR.

$x(\theta) = R[(\pi-\theta) \sin\theta + 1 - \cos\theta)]$

$y(\theta) = R[(\pi-\theta) \cos\theta + \sin\theta]$

"Transforming to integrate over θ instead of y, and using the Jacobian $|dy/d\theta| = R|\theta - \pi| \sin\theta$,

$V_2 = 2\pi R^3 \int_\pi^0 d\theta\, (\theta - \pi) \sin\theta[(\pi - \theta) \cos\theta + \sin\theta][(\pi - \theta) \sin\theta + 1 - \cos\theta)]$

"This integral can easily be evaluated by the online integrator from Mathematica, for example, but I chose just for fun to do it by hand. Either way the answer is

$V_2 = 2\pi R^3({}^3/_2\, \pi^2 - {}^{16}/_3)$

"Subtracting from V_2 the area of the satellite, and adding in the area of the hemisphere V_1, results in the total area accessible by the astronaut:

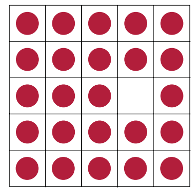
$V_{tot} = \pi R^3({}^2/_3\pi^3 + 3\pi^2 - 12)$."

N/D 3. Perhaps to balance our increase in dimensions in the previous problem, Rocco Giovanniello has lowered his three-dimensional "wink problem" to a mere two dimensions.

Consider a five-by-five checkerboard with 24 of the squares containing a wink each; the remaining square is empty. Using up-down and left-right jumps, can you remove winks until only one remains? The specific problem posed uses the starting configuration below and permits the one remaining wink to be on any square.

Chris Brooks sent the following solution, in which moves are for pieces designated by columns one to five, from the left, and rows one to five, from the bottom:

(1) 2,3 to 4,3	(13) 3,4 to 3,2
(2) 5,3 to 3,3	(14) 1,4 to 3,4
(3) 2,1 to 2,3	(15) 5,4 to 5,2
(4) 2,3 to 4,3	(16) 3,5 to 3,3
(5) 4,1 to 2,1	(17) 1,5 to 3,5
(6) 1,1 to 3,1	(18) 3,2 to 3,4
(7) 4,2 to 2,2	(19) 3,4 to 5,4
(8) 1,2 to 3,2	(20) 5,5 to 5,3
(9) 3,1 to 3,3	(21) 3,5 to 5,5
(10) 4,3 to 2,3	(22) 5,2 to 5,4
(11) 1,3 to 3,3	(23) 5,5 to 5,3
(12) 5,1 to 5,3	



BETTER LATE THAN NEVER

2007 S/O 1. Tom Terwilliger asserts that the 64 possible distributions are not equally likely and that the resulting chance of succeeding is 67.3 percent.

2007 N/D 1. Terwilliger asserts that this problem made the same unwarranted assumption and that, when it's corrected, the probability of a favorable diamond split is 96.27 percent, which is less than the 96.39 percent chance that three hearts can be cashed.

2008 J/A 2. Mark Fineman believes that, although true, it is not obvious that placing children far apart allows no three to be collinear and all distances to be unique.

OTHER RESPONDERS

Responses have also been received from S. Berger, R. Bird, P. Bonomo, D. Dechman, J. Feil, J. Hardis, T. Harriman, L. Kyser, A. Ornstein, J. Roy, E. Sard, L. Schaider, A. Seckinger, E. Sheldon, L. Sonn, C. Swift, and R. Wake.

PROPOSER'S SOLUTION TO SPEED PROBLEM

Mile markers on interstate highways increase traveling north and restart when crossing a state line. The car traveled 300 miles, so it must have crossed a state line at mile marker 150 in the first state. ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York NY 10003, or to gottlieb@nyu.edu.