

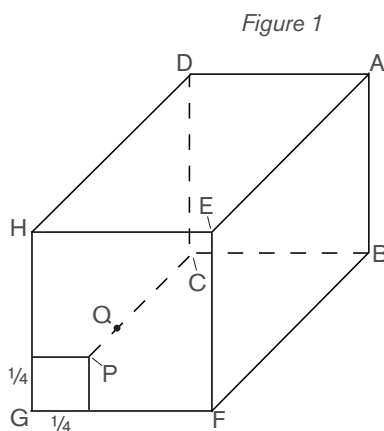
“As far as I can tell, this setup satisfies the constraints of the problem. If he picks the first three key rings, then he is out of luck, so he has to pick four. Clearly, any four key rings will work.

“Note that at first glance it appears that we could omit keys A5 through A12 entirely and still have a valid solution; however, in that scenario, the man could distinguish ring five from the rest because it has four keys instead of two keys, and so he could adopt the strategy of always picking the ring with four keys and one other ring. Introducing the locks A5 through A12 makes the rings indistinguishable from each other.

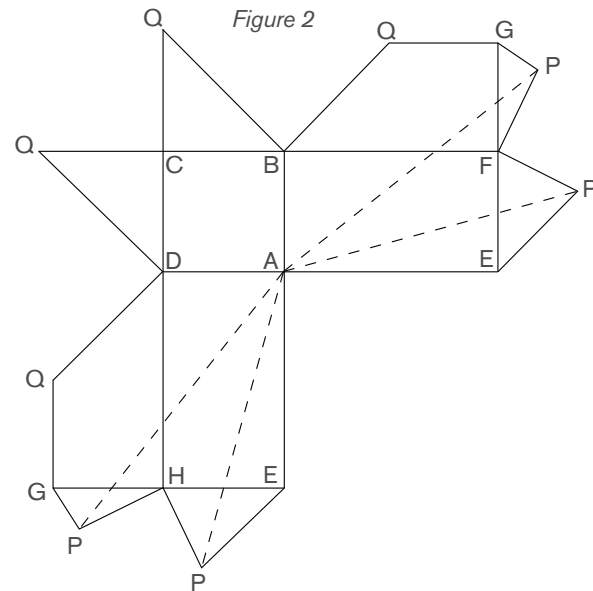
“To show that there is no configuration that requires the man to take all five rings, suppose toward a contradiction that such a configuration exists. Then there must exist a subset of four rings that omits at least one key for each door. It follows that the fifth (unchosen) ring has at least one key for every door. But the problem stipulates that every ring has at least one key to the locks of two different doors, so if you compare any of the first four rings with the fifth ring, there will be at least two doors having keys on both rings, which violates another stipulation of the problem.

“Now, as I remarked before, this might not be the intended solution. If every door has *exactly* two locks, then a nice graph-theoretic argument shows that three rings suffice: create a vertex for each ring and join two vertices with an edge if the rings have keys to the same door; then the conditions of the puzzle force the graph to be a five-cycle, and to make sure that you pick both ends of some edge, it suffices to pick any three vertices. This is my guess as to the intended problem. However, the problem as stated admits the solution I presented above.”

M/A 3. Consider a bug at a corner of a $1 \times 1 \times 2$ solid. Clearly, the farthest-away point is the diagonally opposite corner, if the bug can travel through the solid. But our bug is restricted to the surface of the solid (vertices, edges, and faces). What point is the farthest away? The extended problem is to find two points that are maximally far apart for the bug.



Charles Wampler claims to have fun with these problems. I am pleased that the results of his fun can be beautifully presented solutions such as the one below:



“For a bug traveling on the surface of a $1 \times 1 \times 2$ rectangular solid, the farthest point from a corner, say A, is not the diagonally opposite corner, say G (see Figure 1). It is a point, say P, on the opposing 1×1 face, $\frac{1}{4}$ the way along the diagonal from G. Interestingly, there are four shortest paths from A to P, all of length $|AP| = \sqrt{65}/8$. This is slightly larger than $|AG| = \sqrt{8}$ (for which there are two paths). Figure 2 (not to scale) shows an unfolding of the surface of the solid onto the plane such that the shortest path from A to any other point is a straight line. (By the way, point Q, the midpoint of edge CG, also has four shortest paths from A, all of length $\sqrt{5}$.)

“The two points of the surface that are maximally far apart are the centers of the 1×1 faces. These are three units apart.”

OTHER RESPONDERS

Responses have also been received from R. Ackenberg, D. Aucamp, D. Boika, M. Boltin, D. Dechman, J. Feil, R. Giovanniello, T. Harriman, S. Harris, R. Haskell, K. Lebensold, Z. Moledina, J. Prussing, K. Rosato, and A. Sood.

PROPOSER'S SOLUTION TO SPEED PROBLEM

12.566.

These are the values where the area equals the perimeter. ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York NY 10003, or to gottlieb@nyu.edu.