PUZZLE CORNER

This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 0, and 8) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2007 yearly problem is in the "Solutions" section.

As an extension of the well-known litany "cosine, secant, tangent, sine, 3 point 1 4 1 5 9," Jacques Kourkene sent a much longer French poem encoding the digits of pi as the number of letters in each word (with an *x* coding for zero). My French was not up to the task of translating it, so my former NYU colleague Ed Schonberg kindly helped out.

Que j'aime à faire connaître un nombre utile aux sages! / Immortel Archimède, artiste, ingénieur, / Qui de ton jugement peut sonder la valeur? / Pour moi, ton problème eut de pareils avantages. / Flute x je t'insulte pourtant bien o!

How I love to make known a number useful to sages! / Immortal Archimedes, artist, engineer, / Who in your judgment can plumb the value? / For me, your problem has some similar advantages. / Damn, I abuse you anyway, O!

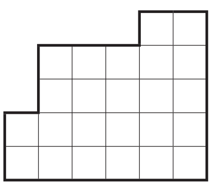
PROBLEMS

Y2008. How many integers from 1 to 100 can you form using the digits 2, 0, 0, and 8 exactly once each; the operators +, -, × (multiplication), and / (division); and exponentiation? We desire solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 0, 8 are preferred. Parentheses may be used for grouping; they do not count as operators. A leading minus sign does count as an operator. Zero to the zero power is not permitted.

J/F 1. An unusual bridge problem in which both declarer and the defenders try to get the minimum number of tricks. Perhaps Larry Kells thought of this because that's how he views his friends' play (or how they view his).

What is the weakest combined holding (high-card points) that N-S can have and still make seven spades with worst play on all sides? What about seven NT?

J/F 2. Consider the following diagram:



It first appeared in the October 1987 *Johns Hopkins Magazine*, in "Golomb's Gambits," by Solomon Golomb. Can you divide the figure into four congruent pieces? There are two solutions.

SPEED DEPARTMENT

Ted Mita is a fan of both New Year's Day and New Year's Eve. He longs for the year in which they both fall on the same day of the week. Can you help him?

SOLUTIONS

Y2007. As has been discussed in the column many times, I do not permit 0^0 (presumably because my training was in continuous rather than discrete mathematics). The following solution is from Burton Rothleder. An asterisk indicates that the digits are used in order. I changed "0 + 0" to "00." If I had previous ruled that 00 is not a legal way to write 0, then please change these back. I don't remember such a ruling and do believe 00 is a valid, operator-free representation for zero.

*$1 = 2^0 + 0 \times 7$	$21 = 20 + 7^0$
*$2 = 2 + 00 \times 7$	*$27 = 20 + 0 + 7$
$3 = 2 + 0 + 7^0$	$35 = 70/2 + 0$
$5 = 00 - 2 + 7$	$49 = 7^2 + 00$
$6 = 7 - 2^{00}$	$50 = 70 - 20$
*$7 = 2 \times 00 + 7$	$68 = 70 - 2 + 0$
*$8 = 2^{00} + 7$	$69 = 70 - 2^0$
*$9 = 2 + 00 + 7$	$70 = 70 - 2 \times 0$
*$13 = 20 - 0 - 7$	$71 = 70 + 2^0$
*$14 = (2 + 00) \times 7$	$72 = 70 + 2 + 0$
$19 = 20 - 7^0$	$90 = 20 + 70$
*$20 = 20 + 0 \times 7$	

S/O 1. There seems to be consensus that if the clubs split very badly, the hand cannot be made against best defense. Frank Model believes he has about a 64.26 percent chance of making the slam with the following play.

"The six outstanding clubs have 64 possible distributions (1 + 6 + 15 + 20 + 15 + 6 + 1). The following line caters to 41.125 of them.

"Ruff the diamond ace and then play the club ace. If everyone follows, play the king. If everyone follows, ruff a diamond high to get back to your hand. Lead a low club. If West shows out, you can ruff low, return to your hand with another high diamond ruff, ruff a club high, return to your hand with a high spade ruff, draw trumps, and claim. If West follows, ruff high. If East also follows, the rest are yours. This covers all 20 situations when clubs are 3-3 and all 15 situations when West has two clubs.

"If West has four clubs, you are still okay if East has the doubleton queen (five situations), as a single high-club ruff sets up the suit.

"If West has just one club, you are okay if it is the queen (one situation), as you can ruff two clubs (as cheaply as necessary) to set up the suit.

"This covers 41 of the 64 possible distributions, but there is still one more chance. If West has five clubs, and East has the singleton queen, you can still make the hand by ruffing two clubs, as long as East does not have a club. I can only estimate this probability, because it depends on whether an opponent shows out during the high ruff back to the South hand. Assuming that everyone shows in, as is most likely by far, at the moment of truth West will have six unknown cards, while East will have ten, so the probabilty of East having neither club is something like $6/16 \times 5/15 = 1/8$. So this line will win in about one-eighth of a 'situation,' using my terminology."

S/O 2. Naomi Markovitz found the minimal, 17-minute solution. First A and B cross; then A (or B) returns; then C and D cross; then B (or, respectively, A) returns; and finally, A and B cross again.

Richard Hess notes that after the third step, all have "crossed the bridge," so you can "solve" the problem using 13 minutes if you permit a trick answer. A different trick answer came from David Cipolla's seven-year-old son, who suggested that A and B walk over and then throw the flashlight back to C and D.

S/O 3. Don Aucamp offers a general procedure that he believes works for many/all such problems (I am deliberately vague on "many/all," since "such problems" is itself vague). Ken Zeger and Google recommend www.math.gatech.edu/news/conferences/reu06abs.html and www.mathpuzzle.com/logicalhats.txt for material on three-hats problems. Finally, Drew McDermott sent a longer analysis that can be found on the "Puzzle Corner" Web page, cs.nyu.edu/~gottlieb/tr.

Aucamp writes that the solution is C = 72 and the other two numbers are 18 and 54. His analysis is as follows:

"The following table is the result of a relatively simple logical procedure in which the solution for each stage is trivially found from the solutions of the prior stages.

"It is easy to show that the sum (A + B + C) must be even, the lowest possible sum is 4, and each number is either the sum of the other two or the positive difference between them. Case 1 in the table gives the results when sum = 4, A = 1, B = 1, and C = 2. In this case, C obviously wins on the first round since both A and B are uncertain whether their numbers are 1 or 3, whereas C knows her number must be A + B = 2. Cases 2 and 3 are similarly solved. Note that the winner in each case is the one with the highest number—which will always happen, since this is the first person who can make profitable use of the table.

case	sum	A	B	C	winner	round
1	4	1	1	2	C	1
2	4	1	2	1	B	1
3	4	2	1	1	A	1
4	6	1	2	3	C	1
5	6	1	3	2	B	2
6	6	2	1	3	C	1
7	6	2	3	1	B	1
8	6	3	1	2	A	2
9	6	3	2	1	A	1
10	8	1	3	4	C	2

"The next higher sum (6) is first solved in case 4, where A = 1, B = 2, and C = 3. In this case, C sees that A = 1 and B = 2, so C is either 1 or 3. However, C notes that B does not declare herself the winner in round one (case 2), so C is not 1, and therefore C = 3. The rest of the table is produced in like manner. Take, for example, case 10, which will turn out to be useful in solving the given problem, where C wins in round two (sum = 8, A = 1, B = 3, C = 4). C knows her number must be either B - A = 2 or A + B = 4. Since B passes in round two, then from case 5 it is clear that C does not equal 2, so C = 4.

"Note that the same logic applies when all A, B, C, and sum values in the entire table are multiplied by a positive integer. When that integer is 18, then case 10 becomes sum = 144, A = 18, B = 54, and C = 72; and case 5 becomes sum = 108, A = 18, B = 54, and C = 36. Now, if C sees A = 18 and B = 54, then either C = B - A = 36 or C = A + B = 72. B passing in round two rules out C = 36 because of case 5, and therefore C knows that A = 18, B = 54, C = 72, and sum = 144. Alternatively, if C sees sum = 144, A = 54, and B = 18, then case 8 rules out C = 36, so C = 72. Thus, in either event, sum = 144, C = 72, and the other two numbers are 18 and 54, which solves the posed problem."

OTHER RESPONDERS

Responses have also been received from E. Chalom, D. Cipolla, G. Coram, C. Dale, B. Edelman, D. Emmes, J. Feil, E. Foster, A. and J. Gail, T. Hafer, J. Hardis, R. Herron, H. Ingraham, L. Iori, W. Jasper, C. Johnson, J. Karlsson, J. Karnofsky, D. Katz, F. Klock, C. Lee, Z. Moledina, A. Ornstein, A. Perez de Lema, Z. Rivkin, M. Rothkopf, A. Sahai, L. Sartori, P. Schottler, W. Seaman, M. Seidel, E. Signorelli, A. Taylor, A. Trojan, and M. Williams.

PROPOSER'S SOLUTION TO SPEED PROBLEM

Except for leap years, they always fall on the same day of the week, since they are 364 days apart (considering both holidays in the same year).